Chapter 12

APPLIED SCALING THEORY

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The increasing use of scale represents an enduring change in the way that ecological research is pursued, rather than another idea in a succession of concepts (often borrowed) that have passed through ecology. Strong interest in the topic is now over a decade old and still burgeoning. Multiscale spatial analysis can be traced back to agricultural trials in the early part of the century (Mercer and Hall 1911), with several notable developments in the 1970s, including recognition of the need for explicit choice of spatial scale in survey design (Wiens 1976; Smith 1978), and the linkage of space and time scales in paleontology (Valentine 1973), terrestrial ecology (Shugart 1978), and aquatic ecology (Haury et al. 1978; Steele 1978a). In the mid 1980s, the rate of publication on spatial scaling expanded rapidly.

The need for multiscale spatial and temporal analysis is now widely recognized. It has become increasingly clear that (1) spatial and temporal patterns depend on the scale of analysis (Platt and Denman 1975; Delcourt et al. 1983), (2) experimental results cannot be extrapolated directly to larger scales (Ricklefs 1987), (3) biological interactions with the environment occur at multiple scales (Harris 1980; O'Neill et al. 1986), (4) population processes do not occur at scales convenient for investigation (Dayton and Tegner 1984), (5) environmental problems arise through propagation of effects across scales, and (6) there is no single or "characteristic" scale for research (Levin 1992).

Verbal and graphical concepts from the 1980s are now evolving into quantitative techniques. Two books (Turner and Gardner 1991a; Schneider 1994a) have appeared, and new techniques continue to be developed (O'Neill and King, chapter 1). There are so many techniques, some highly complex, that no investigator is able to master them all. This chapter suggests a generic approach that is accessible to any ecologist.

This generic approach begins with a careful definition of scaled quantities, which can be rescaled either isometrically or allometrically. The concept of allometric rescaling is extended from mass-based power laws to less familiar length- and time-based power laws. The concept of scope arises naturally from these power laws, and it proves to be widely useful in applying multiscale analysis to practical problems. Examples of application include scope diagrams for natural phenomena, measurement instruments, surveys, and experiments. The principle of homogeneity of scope provides a link between scaling theory and statistical analysis, and it also proves useful in linking experiments (typically at a small scale) to theory (typically at a larger scale).

Scaled Quantities

Ecologists work with definable quantities, not with pure numbers or mathematical abstractions divorced from measurement. The distinction between a quantity and a number might appear to be inconsequential, but it is not. The mathematical rules for working with scaled quantities differ from those for working with numbers. When a number ($N = 78$) replaces a quantity such as world population size—78 *Grus americana* (whooping crane) in 1981—scale is lost.

A well-defined quantity has five parts: (1) a name, (2) a symbol, (3) a procedural statement that prescribes the conditions for measurement, or for calculation from measurement, (4) a set of numbers generated by the procedural statement, and (5) units on one of several types of measurement scales. A measured quantity is conveniently represented as a symbol that equals a set of numbers arranged inside brackets, multiplied by the unit of measurement. An example is the length of *Gadus morhua* (codfish) upon settlement out of the plankton into benthic habitats:

Procedural statement	Name	Symbol	Numbers	•	Units
Length, snout to caudal peduncle	Standard length	sL	$\begin{bmatrix}55 \\ 83 \\ 48\end{bmatrix}$	•	mm

Units occur on one of the four types of measurement scales defined by Stevens (1946). On a nominal scale, the units are "yes" or "no" (present or absent, 0/1, etc.). On an ordinal scale, the unit is the rank of an object. On an interval scale, the unit is a count of the number of units that separate one measurement from another. For interval scale measurement, the textbook example is temperature in degrees Celsius: the zero point is arbitrary. On a ratio scale, units are counted from a natural zero point; the textbook example for this scale is temperature in degrees Kelvin. Both ratio scale (e.g., mass) and interval scale quantities (e.g., degrees Celsius) are used in rescaling.

Allometric Rescaling

Any quantity can be rescaled according to its similarity to another quantity. The rescaling is isometric if a direct proportion is used. For example, the volume of an animal scales isometrically with mass because animals are composed largely of water, which is nearly incompressible. Consequently, a rescaling of the volume (say, by a factor of 2) will rescale the mass by the same factor. The idea of isometric rescaling can be expressed as a proportion:

$$\frac{Volume_{big}}{Volume_{small}} = \frac{Mass_{big}}{Mass_{small}} \tag{1}$$

Allometric rescaling occurs if the proportion is other than direct. The volume of an organism will be proportional to the volume and hence to the cube of the length, not to length itself. A rescaling of length (say, by a factor of 2) will change the volume by a factor of 2^3. The idea of allometric rescaling is expressed as a proportion modified by an exponent:

$$\frac{Volume_{big}}{Volume_{small}} = \left(\frac{Length_{big}}{Length_{small}}\right)^3 \tag{2}$$

Another way to look at this is that for isometric rescaling, the scaling exponent is unity, whereas for allometric rescaling, the exponent differs from unity.

Allometry usually refers to a special case of allometric rescaling in which function or form has been scaled to body size. An example is respiration, which scales as $(Mass)^{0.7}$. Allometric rescaling according to body size was developed by D'Arcy Thompson in his 1917 treatise *On Growth and Form* (Thompson 1961). This landmark book in biology illustrates the principle of similitude with examples based largely on body size and geometric similarity. Thompson advocated the more general principle of similitude, which includes geometric, hydrodynamic, thermal, and other forms of similarity. Recent authors (Peters 1983; Schmidt-Nielsen 1984) have tended to expand the term toward its root meaning, which refers to metrics other than body size.

The steps in allometric rescaling are:

1. State the conditions under which two quantities are considered similar.
2. Express similarity as a proportion. The generic expression rescales a quantity Q according to similarity to another quantity Y:

$$\frac{Q_{new}}{Q_{old}} = \left(\frac{Y_{new}}{Y_{old}}\right)^{\beta} \qquad (3)$$

3. Rearrange to permit calculation of the rescaled quantity Q_{new} from Q_{old} according to the allometric rescaling factor $(Y_{new}/Y_{old})^{\beta}$.

$$Q_{new} = Q_{old}\left(\frac{Y_{new}}{Y_{old}}\right)^{\beta} \qquad (4)$$

The rescaling factor is a ratio (Y_{new}/Y_{old}) raised to the power β. This ratio will be called the scope of the rescaling. Y can be any quantity. The most common is mass, for which there is extensive literature (Peters 1983). Allometric rescaling according to length has become increasingly common (Sugihara and May 1990).

Here is an example of allometric rescaling according to body mass. Maximum running speed ($Vmax = \mathrm{m} \cdot \mathrm{s}^{-1}$) does not increase isometrically with body mass; the scaling exponent as estimated by Bonner (1965) is $\beta = 0.38$, and hence

$$\frac{Vmax_{big}}{Vmax_{small}} = \left(\frac{M_{big}}{M_{small}}\right)^{0.38} \qquad (5)$$

Rearranging this expression results in an equation for calculating the rescaled quantity:

$$Vmax_{big}(M_{small} / M_{big})^{0.38} = Vmax_{small} \qquad (6)$$

$$2\,\mathrm{m\,s^{-1}}\,(1/2)^{0.38} \Rightarrow 1.5\,\mathrm{m\,s^{-1}}$$

The following calculation is for a halving of body mass:

$$(M_{small}/M_{big} = 1/2 = 1\mathrm{kg}/2\mathrm{kg} = 1\mathrm{g}/2\mathrm{g} = \ldots)$$

At half the body mass, speed is rescaled downward by $(2 - 1.5)/2 =$ 25 percent, rather than by half.

Here is an example of allometric rescaling according to length. In a mosaic habitat such as a grassland, the area of bare soil does not scale isometrically with total area. Wiens and Milne (1989) estimate the scaling exponent to be 1.8 and hence

$$\frac{Asoil_{big}}{Asoil_{small}} = \left(\frac{L_{big}}{L_{small}}\right)^{1.8} \qquad (7)$$

This expression says that the area of bare soil ($Asoil = \mathrm{m}^2$) scales with length ($L = m$) raised to fractional power (1.8), rather than with total area (L^2). Because the exponent relative to length is not an integer, it is called a fractal dimension. The physical interpretation of this fractal scaling is that barren soil occurs as a rambling network that becomes less evident at larger scales. Thus, doubling the measurement scale L will not, on average, double the area of bare soil.

As before, this expression is rearranged to obtain an equation for calculating the rescaled quantity:

$$Asoil_{small}(L_{big}/L_{small})^{1.8} = Asoil_{big} \qquad (8)$$

$$2\,\mathrm{m}^2\,(2/1)^{1.8} \Rightarrow 6.96\,\mathrm{m}^2$$

Three Scaling Relations

The idea of allometric rescaling is often expressed in the form of a power law. For example, allometric rescaling according to body mass is typically written as

$$Q = kM^{\beta} \qquad (9a)$$

In this abbreviated expression, Q stands for any quantity, and M stands for body mass. The more complete expression is

$$Q(M) = Q(M_o)(M/M_o)^\beta \tag{9b}$$

Translation to the abbreviated version occurs by writing $Q(M)$ as Q and replacing $Q(M_o) \cdot M_o^{-\beta}$ with k. The best known allometric scaling according to body mass is Kleiber's law, which states that energy use at rest (BMR = joules $\cdot$ s^{-1}) scales allometrically with body mass (M = kg), according to an exponent less than one:

$$BMR = k\ M^\beta \tag{10}$$

This expresses, in quantitative terms, the idea that large organisms live less intensely than small ones, which consume more oxygen and food per unit of body mass (Kleiber 1961).

In a similar fashion, allometric rescaling according to Euclidian length L can be written as

$$Q = k\ L^\beta \tag{11a}$$

where Q again stands for any quantity of interest. D ($= 1 - \beta$) either represents Euclidian dimensions ($D = 1, 2, 3$) or it represents fractal dimensions (Mandelbrot 1977), as in the example of soil area. The fractal dimension $1 < D < 2$ represents a convoluted line embedded in a plane. The fractal dimension $2 < D < 3$ represents a convoluted area embedded in a volume. In this expression, k stands for $Q(L_o) \cdot L_o^{-D}$ and the expanded expression for partial allometry is

$$Q(L) = Q(L_o)(L/L_o)^\beta \tag{11b}$$

Many quantities scale allometrically with time. Examples are frequency of measles epidemics (Sugihara and May 1990), daily rainfall (Lovejoy and Shertzer 1986), and the frequency at which animals change their direction of movement (Frontier 1987). The general abbreviated expression for scaling with time is

$$Q = kT^\tau \tag{12a}$$

where Q again represents any quantity of interest. T is time and τ is an exponent expressing the degree of acceleration ($\tau > 1$) or dampening ($\tau < 1$) as the scope of T increases. In this expression, k stands for $Q(T_o) \cdot T_o^{-\tau}$. The full expression is

$$Q(T) = Q(T_o)(T/T_o)^\tau \tag{12b}$$

The examples to date from the ecological literature suggest that τ is often fractal; environmental processes rarely occur according to the ticking of a clock ($\tau = 1$) or as smooth Newtonian acceleration or deceleration ($\tau = 2$). They occur instead as episodic outbursts, with rates somewhere between the regularity of a clock and Newtonian acceleration.

Mass, length, and time are most commonly encountered in allometric rescaling, but in principle any quantity can be used. One could, for example, rescale organism form and function according to energy exchange, for which the units are energy/time (e.g., watt = joule $\cdot$ s^{-1}):

$$Q(E) = Q(E_o)(E/E_o)^\gamma \tag{13}$$

Allometric rescaling can be applied to complex quantities. An example is scaling of spatial heterogeneity with length scale. If we measure spatial heterogeneity (as a variance) of numbers of organisms (N) in contiguous quadrats of 1 cm, then recompute the variance using contiguous quadrats of 10 cm, we can expect the variance to increase if organisms are clumped. In aquatic habitats, we expect heterogeneity to increase in a regular way with increase in quadrat size, reflecting the physical structure of the surrounding medium (Platt and Denman 1975). This regularity can be expressed as an allometric scaling:

$$\mathrm{Var}(N) = \mathrm{Var}(N_o)(L/L_o)^\beta \tag{14}$$

Spatial allometry ($\beta \neq 1$) forces a rethinking of the use of statistical techniques based on constant variances.

Scaling relations could be written for any pair of quantities thought to be similar. The generic expression for allometric rescaling of some quantity Q according to some other quantity Y is:

$$Q(Y) = Q(Y_o)(Y/Y_o)^\beta \tag{15a}$$

This is easily rewritten as a statement of proportion, as in the generic recipe for allometric rescaling listed above. The short form of the generic scaling law is

$$Q = k\,Y^{\beta} \qquad (15b)$$

Scope

The ratio Y/Y_o was defined in passing as the scope of the rescaling. This ratio is a dimensionless number: it has neither units (e.g., kg, m, s) nor dimensions (e.g., mass, length, time). This lack of units results from the rules for operations on units; it should not be confused with the practice of omitting units. The concept of scope as a dimensionless ratio can be extended to other applications of scaling theory. Several examples, selected from a larger collection (Schneider 1994a), will be described.

The biological and physical phenomena that ecologists study typically have upper and lower limits in space and time. In common usage, the scale of these phenomena refers either to the upper or lower limit. Equivalent pairs of terms (Sugihara and May 1990) are the *minimum* (or inner) scale and the *maximum* (or outer) scale. Still another pair is *grain* and *extent* (Wiens 1989). The scope of a natural phenomenon is defined as the ratio of the upper to the lower limit. Scope is thus the ratio of the extent to the grain, or of the outer to the inner scale. An example is the frequency of El Niño events. The time between events, on average, is 5 years. The temporal scope is $T/T_o = 8$ years ÷ 2 years = 4.

The spatial and temporal scopes of natural phenomena are often graphed in two dimensions. Figure 12.1 shows an example for El Niño events. Logarithmic axes are used because these show multiplicative changes (such as change in ratio), in contrast to linear axes that show additive changes. The upper and lower limits of the hatched area correspond to the upper and lower frequency of such events. The right and left limits of the hatched area correspond to the spatial range and resolution. The distance between upper and lower limits on this logarithmic plot corresponds to the temporal scope T/T_o. The distance from left to right indicates the spatial scope L/L_o. The larger the hatched area in these diagrams, the greater the scope.

Scope (as a dimensionless ratio) can also be used to express the capability of measurement instruments. The scope of an instrument is defined as the ratio of the maximum measurement to the resolution.

A surveyor's chain, for example, has a scope of approximately 10 m ÷ 0.002 m = 50,000. A satellite positioning system (Vande Castle, chapter 13) has a scope of approximately 20,000 km ÷ 0.1 km = 200,000. This is calculated as the maximum distance between two points on the earth's surface, divided by the resolution. The scope of the satellite system is of course greater, but only by a factor of 4. Intuitively, one would have thought that the satellite positioning system would have hundreds of times the scope of a surveyor's chain.

Still another application is the spatial scope of a survey, defined as the ratio of the area to be surveyed to the area of each measurement. An example is the distribution of *Pinus edulis—Juniperus monosperma* (piñon pine-juniper) woodland in a 100,000-ha reserve (Milne et al. 1992). The spatial scope of an aerial survey, based on a resolution

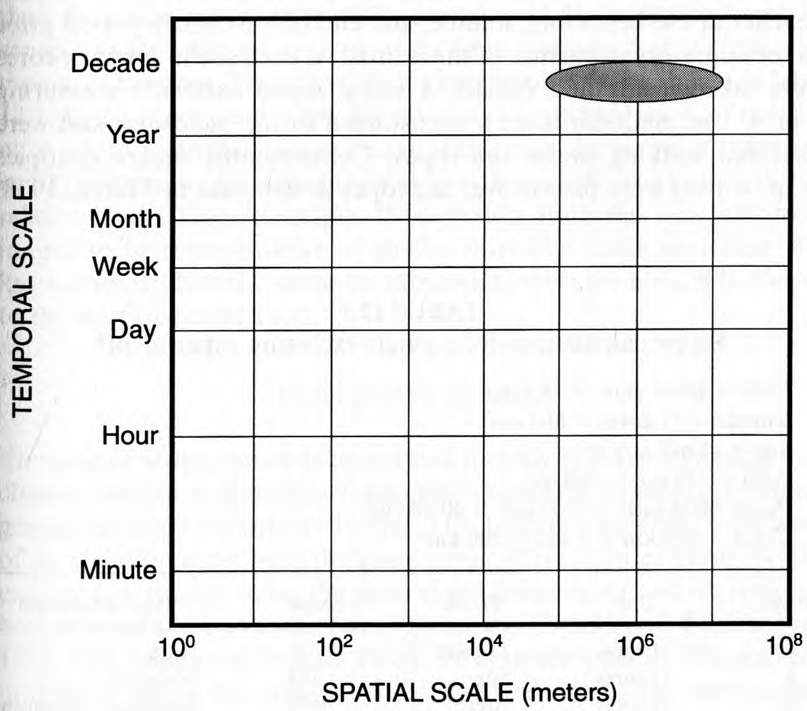

FIGURE 12.1
Scope diagram for El Niño events. Upper limit of *hatched area* is maximum time between events, lower limit is minimum time between events. Left limit is minimum linear extent of affected area. Right limit is maximum linear extent of affected area.

of 2.54 m × 2.54 m pixels in photographs, is 1.6×10^8. The scope of the ground-based survey of 30 randomly selected plots, each 10 m on a side, is 10^5. The scope of the aerial survey is more than 1000 times greater than the 30-plot survey. Computing, then plotting the scope of a survey relative to suspected sources of variation in the quantity of interest, often proves to be informative. Expressed as a graph, scope calculations illustrate the advantages and limitations of a particular instrument for investigating natural phenomena.

The scope of an experiment is a related application. Specifically, the scope of an experiment is the area in which the experiment is set, relative to the minimum area of a sample. Table 12.1 shows a series of scope calculations for an exclosure experiment carried out on a muddy intertidal flat at Punta Mala, Panama (Schneider 1985). To test for effects of avian predation on density of tropical infauna in Panama, invertebrate density was measured at 21 sites distributed over four beaches at the beginning, middle, and end of a 3-month period prior to the migratory departure of shorebirds. At each site in January, cores were taken inside and outside a single roped exclosure measuring 1 m × 1 m. Shorebirds grew accustomed to the exclosures and were observed walking under the ropes. Consequently, square canopies (1 m × 1 m) were placed over the ropes at two sites in March, 1978. This successfully excluded birds. Canopies were placed in areas heavily used by foraging birds. Core samples 10 cm in diameter were taken at all 21 sites in January, March, and April. Cores were taken haphazardly within staked areas and within 1 m of the roped or canopied area.

TABLE 12.1
Scope calculations for a single exclosure experiment*

Core = $\pi(^{10}/_2\ \text{cm})^2 = 78.5\ \text{cm}^2$
Sample = 11 cores = $864\ \text{cm}^2$
Site = $(3\ \text{m})^2 = 9\ \text{m}^2$
Area = $(20\ \text{m})^2 = 400\ \text{m}^2$
Punta Mala inlet = $(200\ \text{m})^2 = 40{,}000\ \text{m}^2$
Coast = 500 km × 1 km = $500\ \text{km}^2$

Design	Unit	Frame	Scope	Type of inference
A	11 cores	11 cores	1	None
B	11 cores	Site	104	Statistical
C	11 cores	Area	4630	Statistical + judgment
D	Site	Area	44	Informal survey
E	Area	Inlet	100	Informal survey
F	Inlet	Coast	12,500	Informal survey

*The frame is the target of inference (population from which the sample is drawn). The scope is the ratio of the area of the frame to the area of the sampling unit.

The scope of the canopy experiment depends on the frame (i.e., the target of inference) and on the sampling unit. The frame and unit correspond (respectively) to the outer and inner scale of the experiment. The inner scale is fixed (a 10-cm diameter core), but there is flexibility (and hence room for judgment) in the choice of outer scale. If the target of inference is the 3 m × 3 m site where cores samples were taken, then the scope is 104 (see table 12.1). This means that a factor of 104 is needed to extrapolate from the sample (11 cores) to the 9-m^2 site from which samples were taken. If the target of inference is the 20 m × 20 m area where the canopy at Punta Mala was placed, then the scope increases to 4630 (see table 12.1). Inference from the sample to this 400-m^2 area is partly from judging that the site was typical of the area. The scope of this judgment is 44, as shown in table 12.1. If the experimental site is judged to be representative of the 200 m × 200 m intertidal area at Punta Mala, then the scope of inference is 100 relative to the experimental area (see table 12.1), and 46,300 relative to the 11-core sample. If the results from this one inlet were judged to be representative of similar intertidal areas stretching 500 km eastward, then the scope or extrapolation factor is 12,500 relative to the inlet (see table 12.1).

Homogeneity of Scope

The idea of scope can be incorporated directly into the statistical machinery used to analyze this experiment, using the principle of homogeneity of scope (Schneider 1994a). This principle says that each term of an equation must have the same scope. Here is an example for the canopy experiment, using the generalized linear model, which includes both normal and nonnormal error structures (Nelder and Wedderburn 1972; McCullagh and Nelder 1989). To evaluate whether the gradient in density across the cage boundary changed during the experiment, the interaction term in a two-way classification of the data must be examined. The generalized linear model relates the response variable (Y = organisms/core) to a random variable ε and a structural model μ composed of explanatory variables.

$$Y = \mu + \varepsilon \qquad (16a)$$

$$\mu = \beta_o + \beta_1 X1 + \beta_2 X2 + \beta_{1\cdot 2} X1 \cdot X_2 \qquad (16b)$$

The explanatory variables are X_1 = presence (0) or absence (1) of canopy, X_2 = beginning (0) or end (1) of experiment, and $X_1 \cdot X_2$ = interaction term, or product of X_1 and X_2. If we assume that the random variable ε is normal (i.e., constant variance around any expected value) in estimating the parameters, then the residuals will not be independent of the fitted values (figure 12.2a). Consequently, an F-distribution cannot be used to calculate Type I error.

The response variable is a count, which suggests that a Poisson error structure might be appropriate. A structural model based on proportions, rather than on differences (as in equation 1), is generally used with Poisson counts:

$$Y = k\, e^{\mu} + \varepsilon \qquad (17a)$$

$$\mu = \beta_o + \beta_1 X1 + \beta_2 X2 + \beta_{1\cdot 2} X1 \cdot X_2 \qquad (17b)$$

This is the statistical model for the familiar two-way contingency test using the G-statistic, the test used originally in the analysis of the experiment (Schneider 1985).

The residuals are still correlated with the fitted values (see figure 12.2b). The variance of the counts exceeds the mean, indicating that the organisms are "clumped" at the scale of a core. Consequently, the assumption for a standard two-way contingency test (i.e., a Poisson error structure) is not appropriate for the data, even though the data are counts. The clumping or overdispersion of the response variable suggests that a Gamma error structure may be appropriate. If we assume a Gamma error structure and use the linear rather than the logarithmic model, we obtain residuals that are independent of the fitted values (see figure 12.2c).

Once we have an acceptable model (equations 16a and 16b with a gamma error structure), we can ask if the gradient in density across the cage boundary changed during the experiment. In statistical terms, should the interaction term be included in the model? The overall deviance of the model from the data is $D = 1.828$ (df = 7), using a Gamma error structure. The deviance of the model from the data, if the interaction term is omitted, is $D = 2.091$ (df = 8). The reduction in deviance $\Delta D = 2.091 - 1.828 = 0.263$, a reduction that is not significant at

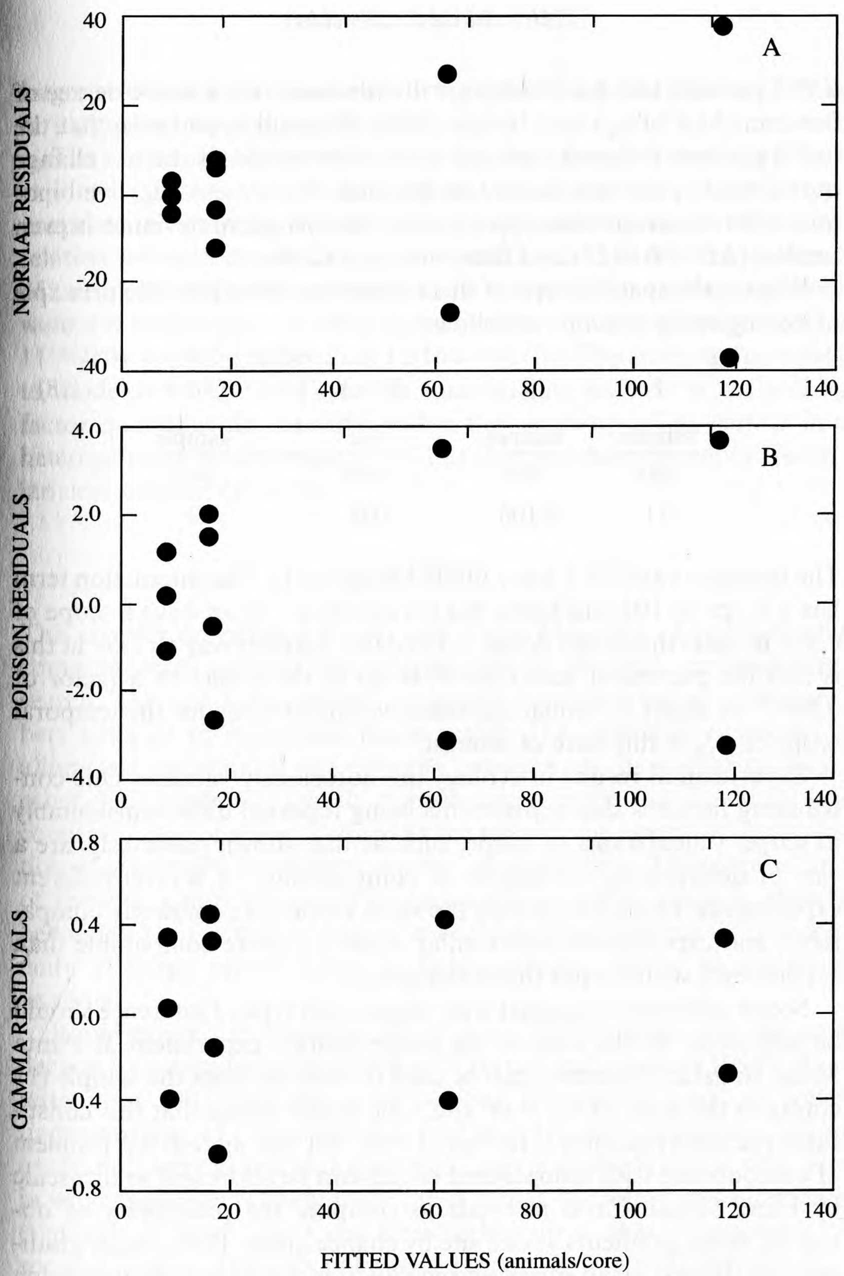

FIGURE 12.2
Residuals from analysis of invertebrate density in core samples, assuming (A) normal errors, (B) Poisson errors, (C) Gamma errors.

$\alpha = 5$ percent, using a Chi-square distribution with a single degree of freedom (McCullagh and Nelder 1989). The null hypothesis, that the radial gradient in density around the canopy boundary did not change during the experiment, cannot be rejected. If we use a negative binomial error structure (also appropriate), the change in deviance is even smaller ($\Delta D = 0.0127$) and hence not significant.

What is the spatial scope of this conclusion? We apply the principle of homogeneity of scope as follows:

$$\begin{array}{cccccccc} Y & = & \beta_{1\cdot 2} & \cdot & X_1 \cdot X_2 & + & \varepsilon & (18) \\ \frac{\text{sample}}{\text{core}} & = & \frac{\text{sample}}{\text{site}} & \cdot & \frac{\text{site}}{\text{core}} & + & \frac{\text{sample}}{\text{core}} & \\ 11 & & 0.106 & \cdot & 104 & & 11 & \end{array}$$

The response variable Y has a limited scope of 11. The interaction term has a scope of 104 and hence the parameter $\beta_{1\cdot 2}$ must have a scope of 0.106 to scale the model down to the data. Another way to look at this is that the parameter scales the data up to the model by a factor of $0.106^{-1,}$ or about 9. Similar calculations can be made for the temporal scope of $\beta_{1\cdot 2,}$ if this were of interest.

Experimental results in ecology are notoriously variable. One contributing factor is that experiments being reported differ considerably in scope. Calculations of scope, such as that shown previously, are a way of determining the degree of comparability of several different experiments. Experiments with the same scope are completely comparable, and experiments with similar scope are more comparable than experiments with scopes that differ greatly.

Scope calculations suggest what degree and type of inference should be employed. In the case of the single canopy experiment at Punta Mala, statistical inference can be used to scale up from the sample (11 cores) to the scale of the 9-m^2 site. One might object that this constitutes pseudoreplication (Hurlbert 1984), but the underlying problem of confounding with unmeasured effects can be addressed at this scale by using Monte Carlo methods to compute the probability of obtaining radial gradients at one site by chance alone. Here, radial gradients are defined as an observed mean within a 1 m × 1 m area being less than the mean in the surrounding 9 m^2 − 1 m^2 = 8 m^2 area. Statistical inference cannot be used to scale up from the sample to the scale of the 400-m^2 area where the canopy was placed, because at this larger scale only one site (experimental unit) was used. Inference to this and larger scales must be based on surveys (Eberhart and Thomas 1992), or judgment, or more experimental units.

The scope calculations shown so far have been isometric, with an exponent of unity. Allometric scalings may well prove to be more appropriate in analyzing the spatial scope of experiments. An allometric relation between density and area would result in a different estimate of the scale-up for the exclosure experiment. If the scaling exponent were 1.8 rather than 2, then the scale-down from model to data is $11^{1.8/2}/104 = 0.083$, rather than $11/104 = 0.106$. The scale-up from data to model is $0.083^{-1} = 12$, rather than 9. This increase in the scaling factor quantifies the intuitive notion that scale-up is less certain in a heterogeneous environment ($D = 1.8$) than in a homogeneous Euclidian environment ($D = 2$).

Summary

This chapter describes a simple and accessible series of concepts and computational methods for applying scaling theory to ecological research. The first key concept uses scaled quantities rather than numbers stripped of units (and hence scale). The second key concept is allometric rescaling of one quantity according to its similarity to another. This concept is readily expressed as a power law. The third key concept is that of scope, defined as the ratio of the range to the resolution of a quantity, instrument, survey, or experiment. The scope of any of these is readily expressed in diagrammatic form, which is already becoming increasingly common in ecology. The principle of homogeneity of scope proves useful in linking scaling theory to statistical analysis.

Rapid expansion of multiscale analysis can be expected in the next decade, as computational machinery (Turner and Gardner 1991b; Rastetter et al. 1992) becomes more familiar. This should lead to a better understanding of environmental problems through improved skill in analyzing physical and biological processes at multiple scales. One area in which progress is likely to be rapid is that of scaling measurements (typically at small space and time scales) up to that of problems of societal interest (typically at larger scales), such as global warming (Innes, chapter 19) or reduced biological diversity. It is now clear that isometric rescaling (the "just multiply" strategy) will not work. Increasing facility in the use of allometric rescaling factors is likely to lead to more accurate computations of the expected effects of human activities on the environment. A second area in which rapid progress seems likely is in the use of fractal spatial scalings. Several interesting applications have already appeared (Lovejoy and Shertzer 1986; Milne et al. 1992), and more are certain to follow.

A third area in which applied scaling theory can potentially increase the effectiveness of ecological practice is in multiscale analysis of the behavior of resource users, and their interaction with the environment (Lee 1993). Humans have always interacted with their environment at multiple spatial scales. The trend in this century has been toward increasing the spatial and energetic scale of environmental alteration (dredging, logging, etc.). Another trend has been increased scale of spatial coordination of resource exploitation. Fish have traditionally been pursued through the uncoordinated activity of individual or recreational fishers. Throughout this century, pursuit has become coordinated at increasingly large scales through technological innovations such as sonar and satellite imagery. The corporations that deploy large fishing fleets are well aware of their ability to locate and harvest every aggregation within extensive areas of the ocean. There has, however, been a lag in the degree to which responsibility for increased resource exploitation has been accepted. Larger-scale institutional arrangements for responsibility have been slow to evolve, leading to overexploitation of renewable resources such as fish and timber. Large-scale responsibility is clearly needed, but large-scale institutional arrangements tend to be rigid and unresponsive (Rykiel, chapter 21). Consequently, environmental science within centrally organized governmental departments may turn out to be irrelevant or wrong. In any case, resource users will consider it irrelevant and wrong (Hobbs, chapter 20). One solution is multiscale or community-based science, in which resource users participate actively in the gathering and assembly of knowledge about those resources. Ecologists involved in conservation-related problems may find that they are more effective working with resource users than they are in relying entirely on technicians. Regardless of the institutional arrangements for ecological research, it is evident that multiscale reasoning is required for understanding the social, legal, and economic settings for ecological problems, as much as for understanding the underlying biological questions.

The concept of scope is based on the principle of similitude, which is widely used in geophysics, oceanography, and meteorology. This principle is used in allometric scaling according to body size, but it has not yet been widely applied in biology at the population or community level. If scaling theory is to become part of normal ecological practice, the first challenge will be the development and mastery of the computational machinery appropriate to applying the principle at the population and community levels of organization. An allied challenge will be integrating this machinery with current statistical methods in ecology. Integration is important because the statistical machinery by itself impedes the use of the principle of similitude. Statistical methods do not distinguish numbers from scaled quantities. This reinforces the widespread (Schneider 1994a) practice of using numbers rather than scaled quantities in both theoretical and experimentally oriented papers in ecology. This practice leads to computational errors because the rules for working with scaled quantities differ from those for working with pure numbers. The practice retards the development of ecological theory by eliminating the mathematical basis for multiscale reasoning. Progress in applying multiscale analysis will accelerate when ecologists become as skilled in applying the principle of similitude as they are now in applying statistical reasoning.

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